

# SO(10) GUT and Quark-Lepton Mass Matrices

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## Abstract

The phenomenological model that all quark and lepton mass matrices have the same zero texture, namely their (1,1), (1,3) and (3,1) components are zeros, is discussed in the context of SO(10) Grand Unified Theories (GUTs). The mass matrices of type I for quarks are consistent with the experimental data in the quark sector. For the lepton sector, consistent fitting to the data of neutrino oscillation experiments force us to use the mass matrix for the charged leptons which is slightly deviated from type I. Given quark masses and charged lepton masses, the model includes 19 free parameters, whereas the SO(10) GUTs gives 16 constrained equations. Changing the remaining three parameters freely, we can fit all the entries of the CKM quark mixing matrix and the MNS lepton mixing matrix, and three neutrino masses consistently with the present experimental data.

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## I. INTRODUCTION

The Downward and upward discrepancy in the atmospheric neutrino deficit in Super Kamiokande [1] together with other neutrino oscillation experiments such as solar neutrino [2], reactor [3] and accelerator [4] experiments drives us to the definite conclusion that neutrinos have masses. These experiments enable us to get a glimpse of high energy physics beyond the Standard Model. In these situations our strategy is as follows. First, we search for the most suitable phenomenological quark and lepton mass matrices which satisfies miscellaneous experiments in the hadron and electro-weak interactions. Next, in order to search for its uniqueness and for its physical implications, such mass matrices are incorporated into the Grand Unified Theories (GUTs). Of course phenomenological mass matrices and GUTs are closely correlated and the real model building is performed going back and forth between these two approaches. Indeed, we consider the seesaw mechanism in neutrino mass matrix [5], which supports minimally SO(10) GUTs. Conversely SO(10) GUTs prefer the mass matrices reflecting some similarity in the quark and lepton sectors. In the seminal work of phenomenological quark mass matrix models [6], Fritzsch proposed a symmetric or hermitian matrices later called a six texture zero model which has vanishing (1,1), (1,3), (3,1) and (2,2) components in both the mass matrices,  $M_u$  for up-type quarks ( $u, c, t$ ) and  $M_d$  for down-type quarks ( $d, s, b$ ). Here n texture zero means that two types of quark mass matrices have totally n zeros in the upper half of hermitian mass matrices, in this case (1,1), (1,3) and (2,2) in each mass matrix. However, this model failed to predict a large top quark mass. Symmetric or hermitian six and five texture zero models were systematically discussed by Ramond et.al. [7]. They found that the hermitian  $M_u$  and  $M_d$  compatible with experiments can have at most five texture zero. Before the work of Ramond et.al. nonsymmetric or non-hermitian six texture zero quark mass matrices model (nearest-neighbor interaction (NNI) model) was proposed by Branco-Lavoura-Mota [8], and Takasugi showed that, by rephasing and rephasing of weak bases, always one of  $M_u$  and  $M_d$  can have the symmetric Fritzsch form and the other does NNI form [9]. Demanding to deal with the quark and lepton mass

matrices on the same footing, we have proposed a four texture zero model [10], in which all the quark and lepton mass matrices,  $M_u, M_d, M_e$  and  $M_\nu$  are hermitian and have the same textures. Here  $M_\nu$  and  $M_e$  are mass matrices of neutrinos ( $\nu_e, \nu_\mu, \nu_\tau$ ) and charged leptons ( $e, \mu, \tau$ ), respectively. Namely their (1,1), (1,3) and (3,1) components are zeros and the others are nonzero valued. This model was also discussed by Du and Xing [11], by Fritzsch and Xing [12], by Kang and Kang [13], by Kang, Kang, Kim, and Kim [14], and by Chkareuli and Froggatt [15], mainly in the quark sector. This model is compatible with the large top quark mass, the small quark mixing angles, and the large  $\nu_\mu$ - $\nu_\tau$  neutrino mixing angles via the seesaw mechanism. In this article, we discuss the above four texture zero model embedding in the SO(10) GUTs. The SO(10) GUTs impose some further constraints on the mass matrices. Using those constraints we predict all the entries of the lepton mixing matrix and neutrino masses, which are consistent with the experimental data, in terms of three free parameters left in the model.

This article is organized as follows. In section 2 we review four texture zero model. In section 3 we present a mass matrix model motivated by SO(10) GUTs. This model is combined with the four texture zero ansatzes in section 4. Section 5 is devoted to summary.

## II. FOUR TEXTURE ZERO QUARK-LEPTON MASS MATRICES

Phenomenological quark mass matrices have been discussed from various points of view [16]. In this section we review our quark and lepton mass matrix model [10]. The mass term in the Lagrangian is given by

$$\begin{aligned}
L_M = & -\overline{q_{R,i}^u} M_{uij} q_{L,j}^u - \overline{q_{R,i}^d} M_{dij} q_{L,j}^d - \overline{l_{R,i}} M_{eij} l_{L,j} - \overline{\nu'_{R,i}} M_{Dij} \nu_{L,j} \\
& - \frac{1}{2} \overline{(\nu_{L,i})^c} M_{Lij} \nu_{L,j} - \frac{1}{2} \overline{(\nu'_{R,i})^c} M_{Rij} \nu'_{R,j} + H.c.
\end{aligned} \tag{1}$$

with

$$q_{L,R}^u = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R}, \quad q_{L,R}^d = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}, \quad l_{L,R} = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{L,R}, \quad \nu_L = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L, \quad \nu'_R = \begin{pmatrix} \nu'_e \\ \nu'_\mu \\ \nu'_\tau \end{pmatrix}_R, \quad (2)$$

where  $M_u$ ,  $M_d$ ,  $M_e$ ,  $M_D$ ,  $M_L$ , and  $M_R$  are the mass matrices for up quarks, down quarks, charged leptons, Dirac neutrinos, left-handed Majorana neutrinos, and right-handed Majorana neutrinos, respectively. The mass matrix of light Majorana neutrinos  $M_\nu$  is given by

$$M_\nu = M_L - M_D^T M_R^{-1} M_D, \quad (3)$$

which is constructed via the seesaw mechanism [5] from the block-diagonalization of neutrino mass matrix,

$$\begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}. \quad (4)$$

We put a ansatz that the mass matrices  $M_u, M_d, M_e$  and  $M_\nu$  are hermitian and have the same textures. Our model is different from the Fritzsch model in the sense that (2,2) components are not zeros and that our model deals with the quark and lepton mass matrices on the same footing. The mass matrices  $M_D$ ,  $M_L$ , and  $M_R$  are, furthermore, assumed to have the same zero texture as  $M_\nu$ . This ansatz restricts the texture forms [10] and we choose the following our texture because it is most closely related with the NNI form [8].

$$\text{NNI : } \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}, \quad \text{Our Texture : } \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}. \quad (5)$$

The nonvanishing (2,2) component distinguishes our form from NNI's. Thus the quark and lepton mass matrices are described as follows.

$$\begin{aligned}
M_u &= \begin{pmatrix} 0 & A_u & 0 \\ A_u & B_u & C_u \\ 0 & C_u & D_u \end{pmatrix}, \\
M_d &= P_d \begin{pmatrix} 0 & A_d & 0 \\ A_d & B_d & C_d \\ 0 & C_d & D_d \end{pmatrix} P_d^\dagger = \begin{pmatrix} 0 & A_d e^{i\alpha_{12}} & 0 \\ A_d e^{-i\alpha_{12}} & B_d & C_d e^{i\alpha_{23}} \\ 0 & C_d e^{-i\alpha_{23}} & D_d \end{pmatrix}, \\
M_e &= P_e \begin{pmatrix} 0 & A_e & 0 \\ A_e & B_e & C_e \\ 0 & C_e & D_e \end{pmatrix} P_e^\dagger = \begin{pmatrix} 0 & A_e e^{i\beta_{12}} & 0 \\ A_e e^{-i\beta_{12}} & B_e & C_e e^{i\beta_{23}} \\ 0 & C_e e^{-i\beta_{23}} & D_e \end{pmatrix}, \\
M_\nu &= \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \end{pmatrix},
\end{aligned} \tag{6}$$

where  $P_d \equiv \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ ,  $\alpha_{ij} \equiv \alpha_i - \alpha_j$ , and  $P_e \equiv \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$ ,  $\beta_{ij} \equiv \beta_i - \beta_j$ .

Let us discuss the relations between the following texture's components of mass matrix  $M$ :

$$M = \begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & D \end{pmatrix} \tag{7}$$

and its eigenmass  $m_i$ . They satisfy

$$\begin{aligned}
B + D &= m_1 + m_2 + m_3, \\
BD - C^2 - A^2 &= m_1 m_2 + m_2 m_3 + m_3 m_1, \\
DA^2 &= -m_1 m_2 m_3.
\end{aligned} \tag{8}$$

Therefore, the mass matrix is classified into two types by choosing  $B$  and  $D$  as follows:

$$\begin{aligned}
[\text{type I}] \quad B &= m_2, \quad D = m_3 + m_1 \\
[\text{type II}] \quad B &= m_1, \quad D = m_3 + m_2
\end{aligned} \tag{9}$$

In the previous paper [10] we showed that type I is compatible with the experimental data both for the quark and lepton mass matrices. So, we concentrate ourselves on the type I case.

In the type I case ( $B = m_2$ ,  $D = m_3 + m_1$ ), the other  $A$  and  $C$  take the following value from Eq.(8)

$$A = \sqrt{\frac{(-m_1)m_2m_3}{m_3 + m_1}}, \quad C = \sqrt{\frac{(-m_1)m_3(m_3 - m_2 + m_1)}{m_3 + m_1}}. \quad (10)$$

Transforming  $m_1$  into  $-m_1$  by rephasing, the mass matrix  $M$  becomes

$$M = \begin{pmatrix} 0 & \sqrt{\frac{m_1 m_2 m_3}{m_3 - m_1}} & 0 \\ \sqrt{\frac{m_1 m_2 m_3}{m_3 - m_1}} & m_2 & \sqrt{\frac{m_1 m_3 (m_3 - m_2 - m_1)}{m_3 - m_1}} \\ 0 & \sqrt{\frac{m_1 m_3 (m_3 - m_2 - m_1)}{m_3 - m_1}} & m_3 - m_1 \end{pmatrix} \simeq \begin{pmatrix} 0 & \sqrt{m_1 m_2} & 0 \\ \sqrt{m_1 m_2} & m_2 & \sqrt{m_1 m_3} \\ 0 & \sqrt{m_1 m_3} & m_3 - m_1 \end{pmatrix}. \quad (11)$$

(for  $m_3 \gg m_2 \gg m_1$ ).

The orthogonal matrix  $O$  which diagonalize  $M$  in Eq.(11) as

$$O^T \begin{pmatrix} 0 & \sqrt{m_1 m_2} & 0 \\ \sqrt{m_1 m_2} & m_2 & \sqrt{m_1 m_3} \\ 0 & \sqrt{m_1 m_3} & m_3 - m_1 \end{pmatrix} O = \begin{pmatrix} -m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad (12)$$

is given by

$$O = \begin{pmatrix} \sqrt{\frac{m_2 m_3^2}{(m_2 + m_1)(m_3^2 - m_1^2)}} & \sqrt{\frac{m_1 m_3 (m_3 - m_2 - m_1)}{(m_2 + m_1)(m_3 - m_2)(m_3 - m_1)}} & \sqrt{\frac{m_1^2 m_2}{(m_3 - m_2)(m_3^2 - m_1^2)}} \\ -\sqrt{\frac{m_1 m_3}{(m_2 + m_1)(m_3 + m_1)}} & \sqrt{\frac{m_2 (m_3 - m_2 - m_1)}{(m_2 + m_1)(m_3 - m_2)}} & \sqrt{\frac{m_1 m_3}{(m_3 - m_2)(m_3 + m_1)}} \\ \sqrt{\frac{m_1^2 (m_3 - m_2 - m_1)}{(m_2 + m_1)(m_3^2 - m_2^2)}} & -\sqrt{\frac{m_1 m_2 m_3}{(m_3 - m_2)(m_2 + m_1)(m_3 - m_1)}} & \sqrt{\frac{(m_3)^2 (m_3 - m_2 - m_1)}{(m_3^2 - m_2^2)(m_3 - m_2)}} \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 & \sqrt{\frac{m_1}{m_2}} & \sqrt{\frac{m_1 m_2^2}{m_3^3}} \\ -\sqrt{\frac{m_1}{m_2}} & 1 & \sqrt{\frac{m_1}{m_3}} \\ \sqrt{\frac{m_1^2}{m_2 m_3}} & -\sqrt{\frac{m_1}{m_3}} & 1 \end{pmatrix} \quad (\text{for } m_3 \gg m_2 \gg m_1). \quad (13)$$

The mass matrices for quarks and charged leptons,  $M_d$ ,  $M_u$ , and  $M_e$  are considered to be of this type I and are given by

$$\begin{aligned}
M_d &\simeq P_d \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b - m_d \end{pmatrix} P_d^\dagger, & M_u &\simeq \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\ 0 & \sqrt{m_u m_t} & m_t - m_u \end{pmatrix}, \\
M_e &\simeq P_e \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau - m_e \end{pmatrix} P_e^\dagger.
\end{aligned} \tag{14}$$

Those  $M_d$ ,  $M_u$ , and  $M_e$  are, respectively, diagonalized by matrices  $P_d O_d$ ,  $O_u$ , and  $P_e O_e$ . Here the orthogonal matrices  $O_d$ ,  $O_u$  and  $O_e$  which diagonalize  $P_d^\dagger M_d P_d$ ,  $M_u$ , and  $P_e^\dagger M_e P_e$  are obtained from Eq. (13) by replacing  $m_1, m_2, m_3$  by  $m_d, m_s, m_b$ , by  $m_u, m_c, m_t$ , and by  $m_e, m_\mu, m_\tau$ , respectively. In this case, the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix  $V$  can be written as

$$V = P_q^{-1} P_d^{-1} O_u^T P_d O_d P_q \simeq \begin{pmatrix} |V_{11}| & |V_{12}| & |V_{13}| e^{-i\phi} \\ -|V_{12}| & |V_{22}| & |V_{23}| \\ |V_{12} V_{23}| - |V_{13}| e^{i\phi} & -|V_{23}| & |V_{33}| \end{pmatrix}. \tag{15}$$

where the  $P_d^{-1}$  factor is included to put  $V$  in the form with diagonal elements real to a good approximation. Furthermore, the  $P_q^{-1}$  and  $P_q = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$  with  $\phi_1 - \phi_2 = \arg(P_d^{-1} O_u^T P_d O_d)_{12}$  and  $\phi_1 - \phi_3 = \arg(P_d^{-1} O_u^T P_d O_d)_{23}$  come from the choice of phase convention as Eq. (15). The explicit forms of the components of  $V$  are obtained [10] as

$$\begin{aligned}
|V_{12}| &\simeq \left| \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} e^{-i\alpha_{12}} \right|, \\
|V_{23}| &\simeq \left| \sqrt{\frac{m_d}{m_b}} - \sqrt{\frac{m_u}{m_t}} e^{-i\alpha_{23}} \right|, \\
|V_{13}| &\simeq \left| \sqrt{\frac{m_d^2 m_s}{m_b^3}} - \sqrt{\frac{m_u}{m_c}} \left( \sqrt{\frac{m_d}{m_b}} - \sqrt{\frac{m_u}{m_t}} e^{-i\alpha_{23}} \right) e^{-i\alpha_{12}} \right| \\
\cos \phi &\simeq \frac{|V_{12}|^2 + m_u/m_c - m_d/m_s}{2|V_{12}| \sqrt{m_u/m_c}}.
\end{aligned} \tag{16}$$

The lepton mixing matrix  $U$  [hereafter we call it the Maki-Nakagawa-Sakata (MNS) mixing matrix [17]], is given by

$$U = P_e^\dagger O_e^T P_e O_\nu = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}, \quad (17)$$

where the  $P_e^\dagger$  factor is included to put  $U$  in the form with diagonal elements real to a good approximation. Here the  $O_\nu$  is the orthogonal matrix which diagonalizes the light Majorana neutrino mass matrices  $M_\nu$  given by Eq.(3).

### III. MASS MATRICES IN THE CONTEXT OF SO(10) GUTS

Even if we succeeded in constructing the quark mass matrices  $M_u$  and  $M_d$  consistent with experiments, we have infinitely many mass matrices equivalent to the  $M_u$  and  $M_d$  which are defined as

$$M'_u = F^\dagger M_u G_u \quad M'_d = F^\dagger M_d G_d, \quad (18)$$

with arbitrary unitary matrices  $F$ ,  $G_u$ , and  $G_d$  in the standard  $SU_L(2) \times U_Y(1)$  model, and with  $G_u = G_d$  in the  $SU_L(2) \times SU_R(2) \times U_Y(1)$  model. The fact that quark and lepton mass matrices have the same form strongly suggests that the quarks and leptons belong to the same multiplets. So in this section we try to incorporate our mass matrix in the context of SO(10) GUTs. We consider two SO(10) symmetry breaking patterns.

$$\begin{aligned} \text{(i)} \quad & SO(10) \rightarrow SU(4) \times SU_L(2) \times SU_R(2) \rightarrow SU_c(3) \times SU_L(2) \times SU_R(2) \times U(1) \rightarrow G_s, \\ \text{(ii)} \quad & SO(10) \rightarrow SU(5) \rightarrow G_s, \end{aligned} \quad (19)$$

where  $G_s = SU_c(3) \times SU_L(2) \times U(1)$ .

#### A. The case of SO(10) breaking down to $SU(4) \times SU_L(2) \times SU_R(2)$

Here we consider the charge-conjugation-conserving (CCC) version [18] [19] [20] of the SO(10) model in which Left-Right discrete (not manifest) symmetry is imposed.



In the  $SO(10)$  model [21] [22] [23], the left- (right-) handed fermions  $\psi_{L(R)i}$  in a given  $i$ -th generation are assigned to a single irreducible **16**. Since  $\mathbf{16} \times \mathbf{16} = \mathbf{10}_S + \mathbf{120}_A + \mathbf{126}_S$ , the fermion masses are generated when the Higgs fields of **10**, and **120**, and **126** dimensional  $SO(10)$  representation (denoted by  $\phi_{10}$ ,  $\phi_{120}$ , and  $\phi_{126}$ , respectively) develop nonvanishing expectation values. Their decomposition under  $SU(4) \times SU_L(2) \times SU_R(2)$  are given by

$$\begin{aligned}\mathbf{10} &= (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}, \mathbf{2}), \\ \mathbf{120} &= (\mathbf{15}, \mathbf{2}, \mathbf{2}) + (\mathbf{6}, \mathbf{3}, \mathbf{1}) + (\overline{\mathbf{6}}, \mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{2}, \mathbf{2}) + (\mathbf{20}, \mathbf{1}, \mathbf{1}), \\ \mathbf{126} &= (\mathbf{10}, \mathbf{3}, \mathbf{1}) + (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3}) + (\mathbf{15}, \mathbf{2}, \mathbf{2}) + (\overline{\mathbf{6}}, \mathbf{1}, \mathbf{1}).\end{aligned}\tag{20}$$

On the other hand, the fermion field of 16-dimensional  $SO(10)$  representation is decomposed as

$$\mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}).\tag{21}$$

With respect to  $SU(4) \times SU_L(2) \times SU_R(2)$ , the left - and right- handed quarks and leptons of a given  $i$ -th generation are assigned as

$$\begin{pmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e \end{pmatrix}_{L(R)} \equiv F_{L(R)1},\tag{22}$$

$F_{L(R)2}$  and  $F_{L(R)3}$  are likewise defined for the 2nd and 3rd generations. Note that their transformation properties are  $F_{Li} = (\mathbf{4}, \mathbf{2}, \mathbf{1})$  and  $F_{Ri} = (\mathbf{4}, \mathbf{1}, \mathbf{2})$  and that  $(F_{Li} + \overline{F_{Ri}})$  yields the **16** of  $SO(10)$ . Since  $(\mathbf{4}, \mathbf{2}, \mathbf{1}) \times (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) = (\mathbf{15}, \mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}, \mathbf{2})$ , the Dirac masses for quarks and leptons are generated when neutral components in a  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  multiplet in  $\phi_{10}$ ,  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  and  $(\mathbf{15}, \mathbf{2}, \mathbf{2})$  in  $\phi_{120}$ , and  $(\mathbf{15}, \mathbf{2}, \mathbf{2})$  in  $\phi_{126}$  of  $SU(4) \times SU_L(2) \times SU_R(2) \subset SO(10)$  develop nonvanishing expectation values. On the other hand, the  $(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})$  and  $(\mathbf{10}, \mathbf{1}, \mathbf{3})$  in  $\phi_{126}$  are responsible for the left- and the right- handed Majorana neutrino masses through the Higgs-lepton-lepton interactions  $(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})(\mathbf{4}, \mathbf{2}, \mathbf{1})(\mathbf{4}, \mathbf{2}, \mathbf{1})$  and  $(\mathbf{10}, \mathbf{1}, \mathbf{3})(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ , respectively. Here the  $(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})$  is the  $SU_L(2)$  triplet Higgs (denoted by  $\phi(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})$ ) and the  $(\mathbf{10}, \mathbf{1}, \mathbf{3})$  is the  $SU_R(2)$  triplet Higgs ( $\phi(\mathbf{10}, \mathbf{1}, \mathbf{3})$ ).

In the CCC version of the SO(10) model , the mass matrices  $M_u$ ,  $M_d$ ,  $M_e$ ,  $M_D$ ,  $M_L$ , and  $M_R$ , for up quarks, down quarks, charged leptons, Dirac neutrinos, left-handed Majorana neutrinos, and right-handed Majorana neutrinos, respectively, are given, in the lowest tree level, by

$$\begin{aligned}
M_u &= S^{(10)}\langle\phi_+^1\rangle + A^{(120)}(\langle\phi_+^3\rangle + \frac{1}{3}\langle\phi_+^{3'}\rangle) + S^{(126)}\frac{1}{3}\langle\phi_+^5\rangle, \\
M_d &= S^{(10)}\langle\phi_-^1\rangle + A^{(120)}(-\langle\phi_-^3\rangle + \frac{1}{3}\langle\phi_-^{3'}\rangle) - S^{(126)}\frac{1}{3}\langle\phi_-^5\rangle, \\
rM_e &= S^{(10)}\langle\phi_-^1\rangle + A^{(120)}(-\langle\phi_-^3\rangle - \langle\phi_-^{3'}\rangle) + S^{(126)}\langle\phi_-^5\rangle, \\
r'M_D &= S^{(10)}\langle\phi_+^1\rangle + A^{(120)}(\langle\phi_+^3\rangle - \langle\phi_+^{3'}\rangle) - S^{(126)}\langle\phi_+^5\rangle, \\
sM_L &= S^{(126)}\langle\phi(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})\rangle, \\
s'M_R &= S^{(126)}\langle\phi(\mathbf{10}, \mathbf{1}, \mathbf{3})\rangle,
\end{aligned} \tag{23}$$

where  $\langle\phi_\pm^1\rangle$  are the vacuum expectation values of the Higgs fields of  $\phi_{10}$ ,  $\langle\phi_\pm^3\rangle$  and  $\langle\phi_\pm^{3'}\rangle$  of  $\phi_{120}$ , and  $\langle\phi_\pm^5\rangle$ ,  $\langle\phi(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})\rangle$  and  $\langle\phi(\mathbf{10}, \mathbf{1}, \mathbf{3})\rangle$  of  $\phi_{126}$ . See Ref [20] for details and the notations. The matrices  $S^{(10)}$  and  $S^{(126)}$  are real symmetric matrices and  $A^{(120)}$  is a pure imaginary matrix. These matrices are the coupling-constant matrices which appear in the Yukawa coupling of fermion fields with Higgs field , which is given by

$$2L_Y = S_{ij}^{(10)}\overline{(\psi_{Li})^c}\phi_{10}\psi_{Lj} + A_{ij}^{(120)}\overline{(\psi_{Li})^c}\phi_{120}\psi_{Lj} + S_{ij}^{(126)}\overline{(\psi_{Li})^c}\phi_{126}\psi_{Lj} + (L \leftrightarrow R) + H.c. \tag{24}$$

The  $\psi_{L(R)i}$  are the 16 irreducible representations of the left- and right- handed fermion fields in a given i'th generation. The property that  $S^{(10)}$  and  $S^{(126)}$  are symmetric and  $A^{(120)}$  is antisymmetric results from the decomposition  $\mathbf{16} \times \mathbf{16} = \mathbf{10}_S + \mathbf{120}_A + \mathbf{126}_S$ , whereas the property that  $S^{(10)}$  and  $S^{(126)}$  are real and  $A^{(120)}$  is pure imaginary is a consequence of their being Hermitian, which in turn comes from the requirement of the invariance of  $L_Y$  under the discrete symmetry  $\psi_L \leftrightarrow \psi_R^c$  [20]. In Eq.(23), the factors  $r \simeq (2 \sim 3)$ ,  $r'$ ,  $s$  and  $s'$ , all roughly of order unity, are the renormalization-group-equation factors [24] [19] which arise from the differences in the renormalization of the lepton and quark masses due to the color quantum numbers of the quarks and so on. The overall factor comes from the loop

correction of gauge boson in the renormalization group equation. Exactly we should consider the evolution equation of Yukawa coupling and in this case mass matrices gets renormalized in somewhat different form. Therefore, this form is an approximation. In this point we will also discuss in the last section.

We now make the following assumptions.

(i) The contribution from **120** is assumed to be small compared with the contributions from **10** and **126**, and hence it is neglected in  $M_u$  and  $M_D$ . On the other hand, it is retained in  $M_d$  and  $M_e$ , for the main term  $S^{(10)}\langle\phi_-^1\rangle$  is smaller by the factor  $\alpha = \langle\phi_-^1\rangle/\langle\phi_+^1\rangle$ , which is of order of  $(m_b/m_t)$  [see Eq.(27)]. This is an assumption for simplicity in order to incorporate Eq.(6)

(ii) All the vacuum expectation values of Higgs fields are assumed to be real so that all the fermion mass matrices are Hermitian.

With these assumptions, Eqs. (24) becomes

$$\begin{aligned}
M_u &= S + \epsilon S', \\
M_d &= \alpha S + S' + A' = \alpha M_u + A' - (\alpha\epsilon - 1)S', \\
rM_e &= \alpha S - 3S' + \delta A' = \alpha M_u + \delta A' - (\alpha\epsilon + 3)S', \\
r'M_D &= S - 3\epsilon S', \\
sM_L &= \beta S', \\
s'M_R &= \gamma S'.
\end{aligned} \tag{25}$$

where the matrices  $S$ ,  $S'$  and  $A'$  and the real parameters  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  are defined by

$$\begin{aligned}
S &= S^{(10)}\langle\phi_+^1\rangle, & S' &= S^{(126)}(-\frac{1}{3}\langle\phi_-^5\rangle), \\
A' &= A^{(120)}(-\langle\phi_-^3\rangle + \frac{1}{3}\langle\phi_-^{3'}\rangle), \\
\alpha &= \langle\phi_-^1\rangle/\langle\phi_+^1\rangle, & \beta &= \langle\phi(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})\rangle/(-\frac{1}{3}\langle\phi_-^5\rangle), \\
\gamma &= \langle\phi(\mathbf{10}, \mathbf{1}, \mathbf{3})\rangle/(-\frac{1}{3}\langle\phi_-^5\rangle), & \delta &= (\langle\phi_-^3\rangle + \langle\phi_-^{3'}\rangle)/(\langle\phi_-^3\rangle - \frac{1}{3}\langle\phi_-^{3'}\rangle).
\end{aligned} \tag{26}$$

Note that solving diagonal elements of Eq.(25) for  $\alpha$ , one finds

$$\alpha = \frac{3\text{Tr}M_d + r\text{Tr}M_e}{3\text{Tr}M_u + r'\text{Tr}M_D} \simeq \frac{m_b}{m_t}, \quad (27)$$

which is about 0.02. As mentioned already, this is why the  $A^{(120)}$  and  $S^{(126)}$  terms are kept in  $M_d$  and  $M_e$ . The Eq.(25) is our  $SO(10)$ -motivated model for fermion mass matrices.

### B. The case of $SO(10)$ breaking down to $SU(5)$

In this case, the fermion masses are also generated when the Higgs fields of **10**, and **120**, and **126** dimensional  $SO(10)$  representation (denoted by  $\phi_{10}$ ,  $\phi_{120}$ , and  $\phi_{126}$ , respectively) develop nonvanishing expectation values. Their decomposition under  $SU(5)$  are given by

$$\begin{aligned} \mathbf{10} &= \mathbf{5} + \bar{\mathbf{5}}, \\ \mathbf{120} &= \mathbf{5} + \bar{\mathbf{5}} + \mathbf{10} + \bar{\mathbf{10}} + \mathbf{45} + \bar{\mathbf{45}}, \\ \mathbf{126} &= \mathbf{1} + \bar{\mathbf{5}} + \mathbf{10} + \bar{\mathbf{15}} + \mathbf{45} + \mathbf{50}. \end{aligned} \quad (28)$$

The yukawa couplings in  $L_Y$  gives the following fermion masses when the neutral components in a **5** and  $\bar{\mathbf{5}}$  Higgs multiplets in  $\phi_{10}$ , **5**,  $\bar{\mathbf{5}}$ , **45**, and  $\bar{\mathbf{45}}$  in  $\phi_{120}$ , and **1**,  $\bar{\mathbf{5}}$ ,  $\bar{\mathbf{15}}$ , and **45** in  $\phi_{126}$  of  $SU(5) \subset SO(10)$  develop nonvanishing expectation values. [25] [23]

$$\begin{aligned} S_{ij}^{(10)} \overline{(\psi_{Li})^c} \phi_{10} \psi_{Lj} &\rightarrow S_{ij}^{(10)} \{ \langle \phi_{10}(\mathbf{5}) \rangle (\overline{u_{R,i}} u_{L,j} + \overline{\nu'_{R,i}} \nu_{L,j}) + \langle \phi_{10}(\bar{\mathbf{5}}) \rangle (\overline{d_{R,i}} d_{L,j} + \overline{e_{R,i}} e_{L,j}) \}, \\ A_{ij}^{(120)} \overline{(\psi_{Li})^c} \phi_{120} \psi_{Lj} &\rightarrow A_{ij}^{(120)} \{ \langle \phi_{120}(\bar{\mathbf{5}}) \rangle (\overline{d_{R,i}} d_{L,j} + \overline{e_{R,i}} e_{L,j}) \\ &\quad + \langle \phi_{120}(\bar{\mathbf{45}}) \rangle (\overline{d_{R,i}} d_{L,j} - 3\overline{e_{R,i}} e_{L,j}) + \langle \phi_{120}(\mathbf{5}) \rangle \overline{\nu'_{R,i}} \nu_{L,j} \\ &\quad + \langle \phi_{120}(\mathbf{45}) \rangle \overline{u_{R,i}} u_{L,j} \}, \\ S_{ij}^{(126)} \overline{(\psi_{Li})^c} \phi_{126} \psi_{Lj} &\rightarrow S_{ij}^{(126)} \{ \langle \phi_{126}(\mathbf{5}) \rangle (\overline{u_{R,i}} u_{L,j} - 3\overline{\nu'_{R,i}} \nu_{L,j}) \\ &\quad + \langle \phi_{126}(\bar{\mathbf{45}}) \rangle (\overline{d_{R,i}} d_{L,j} - 3\overline{e_{R,i}} e_{L,j}) + \langle \phi_{126}(\mathbf{1}) \rangle \overline{\nu_{R,i}^c} \nu'_{R,j} \\ &\quad + \langle \phi_{126}(\mathbf{15}) \rangle \overline{\nu_{L,i}^c} \nu_{L,j} \}, \end{aligned} \quad (29)$$

Therefore, the mass matrices  $M_u$ ,  $M_d$ ,  $M_e$ ,  $M_D$ ,  $M_L$ , and  $M_R$ , for up quarks, down quarks, charged leptons, Dirac neutrinos, left-handed Majorana neutrinos, and right-handed Majorana neutrinos, respectively, are given by

$$\begin{aligned}
M_u &= S^{(10)} \langle \phi_{10}(\mathbf{5}) \rangle + A^{(120)} \langle \phi_{120}(\mathbf{45}) \rangle + S^{(126)} \langle \phi_{126}(\mathbf{5}) \rangle, \\
M_d &= S^{(10)} \langle \phi_{10}(\overline{\mathbf{5}}) \rangle + A^{(120)} (\langle \phi_{120}(\overline{\mathbf{5}}) \rangle + \langle \phi_{120}(\overline{\mathbf{45}}) \rangle) + S^{(126)} \langle \phi_{126}(\overline{\mathbf{45}}) \rangle, \\
rM_e &= S^{(10)} \langle \phi_{10}(\overline{\mathbf{5}}) \rangle + A^{(120)} (\langle \phi_{120}(\overline{\mathbf{5}}) \rangle - 3\langle \phi_{120}(\overline{\mathbf{45}}) \rangle) - 3S^{(126)} \langle \phi_{126}(\overline{\mathbf{45}}) \rangle, \\
r'M_D &= S^{(10)} \langle \phi_{10}(\mathbf{5}) \rangle + A^{(120)} \langle \phi_{120}(\mathbf{5}) \rangle - 3S^{(126)} \langle \phi_{126}(\mathbf{5}) \rangle, \\
sM_L &= S^{(126)} \langle \phi_{126}(\mathbf{15}) \rangle, \\
s'M_R &= S^{(126)} \langle \phi_{126}(\mathbf{1}) \rangle,
\end{aligned} \tag{30}$$

These mass matrices reduce to the same form as Eq.(25) by assuming again that the contributions from **120** Higgs in  $M_u$  and  $M_D$  are negligible and by defining the matrices  $S$ ,  $S'$ , and  $A'$  and the real parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ , instead of Eq.(26), as

$$\begin{aligned}
S &= S^{(10)} \langle \phi_{10}(\mathbf{5}) \rangle, & S' &= S^{(126)} \langle \phi_{126}(\overline{\mathbf{45}}) \rangle, \\
A' &= A^{(120)} (\langle \phi_{120}(\overline{\mathbf{5}}) \rangle + \langle \phi_{120}(\overline{\mathbf{45}}) \rangle), \\
\alpha &= \langle \phi_{10}(\overline{\mathbf{5}}) \rangle / \langle \phi_{10}(\mathbf{5}) \rangle, & \beta &= \langle \phi_{126}(\mathbf{15}) \rangle / \langle \phi_{126}(\overline{\mathbf{45}}) \rangle, \\
\gamma &= \langle \phi_{126}(\mathbf{1}) \rangle / \langle \phi_{126}(\overline{\mathbf{45}}) \rangle, \\
\delta &= (\langle \phi_{120}(\overline{\mathbf{5}}) \rangle - 3\langle \phi_{120}(\overline{\mathbf{45}}) \rangle) / (\langle \phi_{120}(\overline{\mathbf{5}}) \rangle + \langle \phi_{120}(\overline{\mathbf{45}}) \rangle),
\end{aligned} \tag{31}$$

Thus, Eq.(25) is our SO(10)-motivated model for fermion mass matrices both for the two SO(10) breaking patterns (i) and (ii) in Eq.(19).

#### IV. FOUR TEXTURE ZERO MODEL IN SO(10)

The SO(10) model Eq.(25) is now combined with the four texture zero ansatzes for  $M_u$ ,  $M_d$  and  $M_e$  which are given by Eq.(6).

First it follows from Eq.(25) that  $S$ ,  $S'$  and  $A'$  are represented in terms of the symmetric (antisymmetric) parts,  $M_{sym}$  ( $M_{antisym}$ ), of  $M_u$ ,  $M_d$  and  $M_e$ ;

$$\begin{aligned}
(1 - \alpha\epsilon)S &= (M_u)_{sym} - \epsilon(M_d)_{sym}, \\
S' &= \frac{1}{4} \{ (M_d)_{sym} - r(M_e)_{sym} \}, \\
A' &= (M_d)_{antisym}.
\end{aligned} \tag{32}$$

We also find the constraints

$$(1 - \alpha\epsilon)r(M_e)_{sym} = 4\alpha(M_u)_{sym} - (3 + \alpha\epsilon)(M_d)_{sym},$$

$$\delta(M_d)_{antisym} = r(M_e)_{antisym}. \quad (33)$$

Using the four texture zero ansatzes for  $M_u$ ,  $M_d$  and  $M_e$  given by Eq.(6), the respective elements of Eq.(33) become

$$(1 - \alpha\epsilon)rA_e \cos \beta_{12} = 4\alpha A_u - (3 + \alpha\epsilon)A_d \cos \alpha_{12},$$

$$(1 - \alpha\epsilon)rB_e = 4\alpha B_u - (3 + \alpha\epsilon)B_d,$$

$$(1 - \alpha\epsilon)rC_e \cos \beta_{23} = 4\alpha C_u - (3 + \alpha\epsilon)C_d \cos \alpha_{23},$$

$$(1 - \alpha\epsilon)rD_e = 4\alpha D_u - (3 + \alpha\epsilon)D_d, \quad (34)$$

$$\delta A_d \sin \alpha_{12} = rA_e \sin \beta_{12},$$

$$\delta C_d \sin \alpha_{23} = rC_e \sin \beta_{23}.$$

In Eq.(34) there are six equations and eight unknown parameters, namely  $\alpha$ ,  $\epsilon$ ,  $\delta$ ,  $\alpha_{12}$ ,  $\alpha_{23}$ ,  $\beta_{12}$ ,  $\beta_{23}$  and  $r$  provided that  $A_u$ ,  $B_u, \dots, D_e$  are given. In the following, we treat  $\cos \alpha_{12}$  and  $r$  as free parameters so that all the other parameters are functions of them. Here we treat  $r$  as a free parameter too, although we know  $r \simeq (2 \sim 3)$ . Let us present the following useful expressions which are derived from Eq(34):

$$\cos \beta_{12} = \left( \frac{B_e D_d - D_e B_d}{B_u D_d - D_u B_d} \right) \left( \frac{A_u}{A_e} \right) - \left( \frac{B_e D_u - D_e B_u}{B_u D_d - D_u B_d} \right) \left( \frac{A_d}{A_e} \right) \cos \alpha_{12},$$

$$\cos \beta_{23} = \left( \frac{B_e D_d - D_e B_d}{B_u D_d - D_u B_d} \right) \left( \frac{C_u}{C_e} \right) - \left( \frac{B_e D_u - D_e B_u}{B_u D_d - D_u B_d} \right) \left( \frac{C_d}{C_e} \right) \cos \alpha_{23},$$

$$\frac{\sin \beta_{23}}{\sin \alpha_{23}} = \left( \frac{A_e C_d}{A_d C_e} \right) \frac{\sin \beta_{12}}{\sin \alpha_{12}},$$

$$\alpha = \frac{r \left( \frac{B_e D_d - D_e B_d}{B_u D_d - D_u B_d} \right)}{r \left( \frac{B_e D_u - D_e B_u}{B_u D_d - D_u B_d} \right) + 1}, \quad \epsilon = \frac{r \left( \frac{B_e D_u - D_e B_u}{B_u D_d - D_u B_d} \right) - 3}{r \left( \frac{B_e D_d - D_e B_d}{B_u D_d - D_u B_d} \right)}, \quad \delta = r \left( \frac{A_e}{A_d} \right) \frac{\sin \beta_{12}}{\sin \alpha_{12}}. \quad (35)$$

Now we discuss the MNS lepton mixing matrix and neutrino masses. The light Majorana neutrino mass matrix  $M_\nu$  is given by Eq.(3), where the Dirac neutrino, left- and right- handed Majorana neutrino mass matrices  $M_D$ ,  $M_L$ , and  $M_R$  are expressed in terms of the entries

of the quarks and charged lepton mass matrices due to the SO(10) constraints and their expressions are given, from Eqs.(25) and (32), by

$$\begin{aligned}
r'M_D &= S - 3\epsilon S' = \begin{pmatrix} 0 & A_D & 0 \\ A_D & B_D & C_D \\ 0 & C_D & D_D \end{pmatrix}, \\
sM_L &= \beta S' = \beta \begin{pmatrix} 0 & A_{S'} & 0 \\ A_{S'} & B_{S'} & C_{S'} \\ 0 & C_{S'} & D_{S'} \end{pmatrix}, \\
s'M_R &= \gamma S' = \gamma \begin{pmatrix} 0 & A_{S'} & 0 \\ A_{S'} & B_{S'} & C_{S'} \\ 0 & C_{S'} & D_{S'} \end{pmatrix},
\end{aligned} \tag{36}$$

where the elements  $A_D, B_D, C_D, D_D, A_{S'}, B_{S'}, C_{S'},$  and  $D_{S'}$  are obtained as

$$\begin{aligned}
A_D &= A_u - \epsilon(A_d \cos \alpha_{12} - rA_e \cos \beta_{12}), \\
B_D &= B_u - \epsilon(B_d - rB_e), \\
C_D &= C_u - \epsilon(C_d \cos \alpha_{23} - rC_e \cos \beta_{23}), \\
D_D &= D_u - \epsilon(D_d - rD_e),
\end{aligned} \tag{37}$$

and

$$\begin{aligned}
A_{S'} &= \frac{1}{4}(A_d \cos \alpha_{12} - rA_e \cos \beta_{12}), \\
B_{S'} &= \frac{1}{4}(B_d - rB_e), \\
C_{S'} &= \frac{1}{4}(C_d \cos \alpha_{23} - rC_e \cos \beta_{23}), \\
D_{S'} &= \frac{1}{4}(D_d - rD_e).
\end{aligned} \tag{38}$$

In the following analysis, we assume that the contribution of  $M_L$  to  $M_\nu$  in Eq. (3) is much smaller than that of the second term so that we have  $M_\nu = M_L - M_D^T M_R^{-1} M_D \simeq -M_D^T M_R^{-1} M_D$ . Then, all the components  $A_\nu, B_\nu, C_\nu,$  and  $D_\nu$  in Eq.(6) are determined,

from Eqs.(36), (37), and (38), as functions of  $\cos \alpha_{12}$  and  $r$  except for the common overall factor  $s'/(r'^2\gamma)$  as.

$$M_\nu = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \end{pmatrix} = -s'/(r'^2\gamma) \begin{pmatrix} 0 & A_D & 0 \\ A_D & B_D & C_D \\ 0 & C_D & D_D \end{pmatrix} \begin{pmatrix} 0 & A_{S'} & 0 \\ A_{S'} & B_{S'} & C_{S'} \\ 0 & C_{S'} & D_{S'} \end{pmatrix}^{-1} \begin{pmatrix} 0 & A_D & 0 \\ A_D & B_D & C_D \\ 0 & C_D & D_D \end{pmatrix}, \quad (39)$$

$$\begin{aligned} A_\nu &= -\frac{s'}{r'^2\gamma} \left( \frac{A_D^2}{A_{S'}} \right), \\ B_\nu &= -\frac{s'}{r'^2\gamma} \left\{ \frac{A_D B_D}{A_{S'}} + C_D \left( \frac{C_D}{D_{S'}} - \frac{A_D C_{S'}}{A_{S'} D_{S'}} \right) \right. \\ &\quad \left. + A_D \left( \frac{B_D}{A_{S'}} - \frac{C_D C_{S'}}{A_{S'} D_{S'}} - \frac{A_D (-C_{S'}^2 + B_{S'} D_{S'})}{A_{S'}^2 D_{S'}} \right) \right\}, \\ C_\nu &= -\frac{s'}{r'^2\gamma} \left\{ \frac{A_D C_D}{A_{S'}} + D_D \left( \frac{C_D}{D_{S'}} - \frac{A_D C_{S'}}{A_{S'} D_{S'}} \right) \right\}, \\ D_\nu &= -\frac{s'}{r'^2\gamma} \left( \frac{D_D^2}{D_{S'}} \right). \end{aligned} \quad (40)$$

Therefore, the neutrino mass ratios  $m_1/m_2$  and  $m_2/m_3$  and hence MNS lepton mixing matrix elements are also determined as functions of  $\cos \alpha_{12}$  and  $r$ . The common overall factor  $s'/(r'^2\gamma)$  is determined by the  $\Delta m^2$  data from neutrino oscillation experiments. The light Majorana neutrino masses are obtained by diagonalizing  $M_\nu$  as

$$O_\nu^T \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \end{pmatrix} O_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \quad (41)$$

For the case in which  $B_\nu, C_\nu, D_\nu \gg A_\nu$  is satisfied, the neutrino masses are approximately expressed in terms of  $A_\nu, B_\nu, C_\nu$ , and  $D_\nu$  as

$$m_1 \simeq -\frac{D_\nu A_\nu^2}{B_\nu D_\nu - C_\nu^2},$$



$$\begin{aligned}
m_2 &\simeq \frac{1}{2}\{B_\nu + D_\nu - \sqrt{(B_\nu + D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}\} \\
&+ \frac{D_\nu^2 + C_\nu^2 - \frac{1}{2}D_\nu\{B_\nu + D_\nu - \sqrt{(B_\nu + D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}\}}{(B_\nu D_\nu - C_\nu^2)\sqrt{(B_\nu + D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}}A_\nu^2, \\
m_3 &\simeq \frac{1}{2}\{B_\nu + D_\nu + \sqrt{(B_\nu + D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}\}. \\
&- \frac{D_\nu^2 + C_\nu^2 - \frac{1}{2}D_\nu\{B_\nu + D_\nu + \sqrt{(B_\nu + D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}\}}{(B_\nu D_\nu - C_\nu^2)\sqrt{(B_\nu + D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}}A_\nu^2.
\end{aligned} \tag{42}$$

The orthogonal matrices  $O_\nu$  which diagonalizes  $M_\nu$  are expressed in terms of the diagonalized masses  $m_1$ ,  $m_2$ , and  $m_3$  and the matrix components  $A_\nu$ ,  $B_\nu$ ,  $C_\nu$ , and  $D_\nu$  as [14]

$$O_\nu = \begin{pmatrix} \frac{A_\nu}{m_1}(O_\nu)_{21} & \frac{A_\nu}{m_2}(O_\nu)_{22} & \frac{A_\nu}{m_3}(O_\nu)_{23} \\ (O_\nu)_{21} & (O_\nu)_{22} & (O_\nu)_{23} \\ \frac{C_\nu}{m_1 - D_\nu}(O_\nu)_{21} & \frac{C_\nu}{m_2 - D_\nu}(O_\nu)_{22} & \frac{C_\nu}{m_3 - D_\nu}(O_\nu)_{23} \end{pmatrix}, \tag{43}$$

with

$$\begin{aligned}
(O_\nu)_{21}^2 &= \frac{1}{\left(\frac{A_\nu}{m_1}\right)^2 + 1 + \left(\frac{C_\nu}{m_1 - D_\nu}\right)^2}, \\
(O_\nu)_{22}^2 &= \frac{1}{\left(\frac{A_\nu}{m_2}\right)^2 + 1 + \left(\frac{C_\nu}{m_2 - D_\nu}\right)^2}, \\
(O_\nu)_{23}^2 &= \frac{1}{\left(\frac{A_\nu}{m_3}\right)^2 + 1 + \left(\frac{C_\nu}{m_3 - D_\nu}\right)^2}.
\end{aligned} \tag{44}$$

It should be remarked that the light neutrino mass matrix  $M_\nu$  itself is out of type I via the seesaw mechanism and that the MNS lepton mixing matrix is obtained from Eqs. (43) and (17). Since  $O_e$  is almost diagonal, the magnitudes of off-diagonal elements are predominated by Eq.(44). Thus the seesaw mechanism changes the form of lepton mixing matrix from that of CKM matrix given by Eq.(16).

Now, by changing the values of the free parameters in our model, we proceed to find the solutions which are consistent with the recent following findings that (i) the atmospheric neutrino oscillation experiment indicates the  $\nu_\mu$ - $\nu_\tau$  large mixing ( $0.28 \lesssim |U_{23}| \lesssim 0.72$  [26]) with  $\Delta m_{23}^2 = m_3^2 - m_2^2 = (1.5 \sim 6) \times 10^{-3} \simeq 3.5 \times 10^{-3} \text{eV}^2$ , and (ii) the solar neutrino experiments imply the MSW small mixing angle solution [27] with  $\Delta m_{12}^2 = m_2^2 - m_1^2 = (4 \sim 10) \times 10^{-6} \text{eV}^2$  and  $\sin^2 2\theta_{12} = (2 \sim 10) \times 10^{-3}$ , or the large mixing angle solution [27] with

$\Delta m_{12}^2 = m_2^2 - m_1^2 \simeq (8 \sim 30) \times 10^{-6} \text{eV}^2$  and  $\sin^2 2\theta_{12} = (0.5 \sim 1)$ . In the following analysis, we transform  $A_e$ ,  $B_e$ ,  $C_e$ , and  $D_e$  in Eq.(6) into  $-A_e$ ,  $-B_e$ ,  $-C_e$ , and  $-D_e$ , respectively by rephasing of the right-handed charged lepton fields.

First assuming that the mass matrices  $M_u$ ,  $M_d$  and  $M_e$  are all of type I, we calculate numerically the MNS lepton mixing matrix  $U$  using the central values for the running quarks and charged leptons masses at  $\mu = m_Z$  [28]:

$$\begin{aligned} m_u(m_Z) &= 2.33_{-0.45}^{+0.42} \text{MeV}, \quad m_c(m_Z) = 677_{-61}^{+56} \text{MeV}, \quad m_t(m_Z) = 181 \pm 13 \text{GeV}, \\ m_d(m_Z) &= 4.69_{-0.66}^{+0.60} \text{MeV}, \quad m_s(m_Z) = 93.4_{-13.0}^{+11.8} \text{MeV}, \quad m_b(m_Z) = 3.00 \pm 0.11 \text{GeV}, \\ m_e(m_Z) &= 0.487 \text{MeV}, \quad m_\mu(m_Z) = 103 \text{MeV}, \quad m_\tau(m_Z) = 1.75 \text{GeV}. \end{aligned} \quad (45)$$

Since the recent atmospheric neutrino oscillation data indicates large value of (2,3) element of  $U$ , ( $0.28 \lesssim |U_{23}| \lesssim 0.72$  [26]), we obtain the allowed region of the parameters space,  $\cos \alpha_{12}$  vs  $r$  space which reproduces a large  $|U_{23}|$ . The result is given in Fig. 1. In this allowed parameter region,  $r \simeq 3$  is automatically satisfied without any fine tuning. However, we have a serious problem that in this allowed parameter space we cannot accommodate the overall factor  $s'/(r'^2 \gamma)$  simultaneously to the data  $\Delta m_{12}^2 = m_2^2 - m_1^2 \lesssim 10^{-4} \text{eV}^2$  (Here we have adopted a rather conservative value. We accept more restrictive ones later.) from solar neutrino oscillation experiments and the data  $\Delta m_{23}^2 = m_3^2 - m_2^2 \simeq 3.5 \times 10^{-3} \text{eV}^2$  from atmospheric neutrino oscillation experiments. Taking deviations from the central values [28] for quarks and charged leptons masses does not resolve this problem. This difficulty is resolved by abandoning the above assumption that the mass matrices  $M_u$ ,  $M_d$  and  $M_e$  are all of type I. Let us assume that  $M_e$  deviates from type I although  $M_u$  and  $M_d$  are of type I. Then, we can accommodate the overall factor  $s'/(r'^2 \gamma)$  simultaneously to both  $\Delta m^2$  data from solar and atmospheric neutrino oscillation experiments.

Next we discuss this new scenario and show that there are solutions consistent with the data. First we represent the deviation from type I as  $B_e = m_\mu(1 + \xi)$ . In this case, the entries of the mass matrix  $M_e$  for charged leptons in Eq.(6) are given, in the unit of eV, as

$$A_e = \sqrt{\frac{m_e m_\mu m_\tau}{m_\tau - \xi m_\mu - m_e}} \simeq 7.1 \times 10^6,$$

$$\begin{aligned}
B_e &= (1 + \xi)m_\mu \simeq 1.02 \times 10^8(1 + \xi), \\
C_e &= \sqrt{(m_e + \xi m_\mu)(m_\tau - (1 + \xi)m_\mu - m_e) \left( \frac{m_\tau - \xi m_\mu}{m_\tau - \xi m_\mu - m_e} \right)} \simeq \sqrt{8.1 \times 10^{14} + 1.7 \times 10^{17}\xi}, \\
D_e &= -m_e - \xi m_\mu + m_\tau \simeq 7.1 \times 10^9,
\end{aligned} \tag{46}$$

and the expressions for  $\epsilon$ ,  $\alpha$  and the elements of the matrices  $S$  and  $S'$  are given by

$$\begin{aligned}
\epsilon &\simeq -\frac{(-r(1 + \xi)m_\mu + 3m_s)m_t - m_c(3m_b - rm_\tau)}{r((1 + \xi)m_b m_\mu - m_s m_\tau)}, \\
\alpha &\simeq -\frac{r((1 + \xi)m_b m_\mu - (1 + \xi)m_d m_\mu - m_s m_\tau)}{m_b m_c + (-rm_\mu - r\xi m_\mu - m_s)m_t - m_c(m_d - rm_\tau)}, \\
S_{12} &\simeq ((rm_\mu + m_s)(\cos \alpha_{12}\sqrt{m_d m_s}(-3m_b m_c - r(1 + \xi)m_\mu m_t) + \\
&\quad \cos \alpha_{12}\sqrt{m_d m_s}(3m_s m_t + rm_c m_\tau) - r(-m_b m_\mu + m_s m_\tau)\sqrt{m_c m_u}))/ \\
&\quad (4rm_s(m_b m_\mu - m_s m_\tau)), \\
S_{22} &\simeq -\frac{(-rm_\mu - m_s)(-r(1 + \xi)m_\mu + 3m_s)m_t}{4r(m_b m_\mu - m_s m_\tau)}, \\
S_{23} &\simeq (-\cos \alpha_{23}\sqrt{m_b m_d}(-rm_\mu - m_s)m_t \\
&\quad (-3m_b m_c - r(1 + \xi)m_\mu m_t + 3m_s m_t + rm_c m_\tau) - \\
&\quad r(-rm_\mu - m_s)m_t((1 + \xi)m_b m_\mu - m_s m_\tau)\sqrt{m_t m_u})/ \\
&\quad (r^2 m_\tau(m_b m_c m_\mu + m_s(\xi m_\mu m_t - m_c m_\tau)) - \\
&\quad r(3m_b^2 m_c m_\mu + 4m_s^2 m_t m_\tau - m_b m_s((4 + \xi)m_\mu m_t + 3m_c m_\tau))), \\
S_{33} &\simeq -\frac{(-rm_\mu - m_s)m_t(3m_b - rm_\tau)}{4r(m_b m_\mu - m_s m_\tau)}, \\
S'_{12} &\simeq \frac{(\cos \alpha_{12}\sqrt{m_d m_s}(rm_\mu + m_s)m_t - r(m_b m_\mu - m_s m_\tau)\sqrt{m_c m_u})}{4m_s m_t}, \\
S'_{22} &\simeq \frac{1}{4}(rm_\mu + m_s), \\
S'_{23} &\simeq -\left( \cos \alpha_{23}\sqrt{m_b m_d} + \frac{r(m_b m_\mu - m_s m_\tau)m_u}{(-rm_\mu - m_s)\sqrt{m_t m_u}} \right) / \\
&\quad \left( -1 + \frac{(m_b m_\mu - m_s m_\tau)((-r(1 + \xi)m_\mu + 3m_s)m_t - m_c(3m_b - rm_\tau))}{(-rm_\mu - m_s)m_t((1 + \xi)m_b m_\mu - m_s m_\tau)} \right), \\
S'_{33} &\simeq \frac{1}{4}(m_b - m_d + rm_\tau).
\end{aligned} \tag{47}$$

The point  $r \simeq 3$  is rather singular in the following sense. As is seen from Eq.(35),  $\epsilon$  becomes small at  $r \simeq 3$ , hence we can not neglect  $\xi$  in this region. That is,  $\epsilon$  is sensitive to small  $\xi$ .

For instance, substituting Eq.(47) into Eqs.(25) we obtain  $M_D$  and  $M_R$  and therefore  $M_\nu$  through Eq.(39). The behaviors of the elements in the neutrino mass matrix  $M_\nu$  are depicted in Fig.2. The large  $\nu_\mu$ - $\nu_\tau$  neutrino mixing appears under the condition that  $B_\nu \simeq D_\nu$  which is realized at  $\xi \simeq 0.02$ . By changing  $\xi$  freely with fixed  $r(=3)$ , we can well reproduce the experimental data as shown in Figs. 3-5, in which the constraints from  $|U_{23}|$ ,  $\Delta m_{12}^2/\Delta m_{23}^2$ , and the both are satisfied, respectively. It is seen from Fig. 5 that by deviating  $B_e$  a little bit from type I ( $\xi \sim 0.01$ ), we can well reproduce the experimental data for the solar neutrino oscillation and atmospheric neutrino deficit. If we relax the condition  $r=3$  and change  $r$  freely as well as  $\cos \alpha_{12}$  and  $\xi$  around the values of the above solutions, we have the larger allowed region as is shown in Fig. 6. In the above allowed regions shown in Figs. 1-5, we have used only the conservative condition for  $\Delta m_{12}^2$  from the solar neutrino experiments, that is, we have not used the constraints of the mixing angle from the solar neutrino oscillation experiments. When we take them into account in addition to the constraints from  $\Delta m_{12}^2$ , we obtain more restrictive allowed region than that of Fig. 6. Under the condition of the small mixing angle solution for solar neutrino experiments, the larger region of  $|\cos \alpha_{12}|$  in Fig. 6 is eliminated and we have the allowed region as is shown in Fig. 7. On the other hand, under the condition of the large mixing angle solution, the smaller region of  $|\cos \alpha_{12}|$  is eliminated and the allowed region is given in Fig. 8. It should be noted that as seen in Figs. 6-8 our model not only satisfies the experimental observations in the lepton sector but also provides the restriction on the CP violation phase,  $\cos \alpha_{12}$ , from the neutrino oscillation experiments. Of course, we can also restrict the other CP violation phases,  $\cos \alpha_{23}$ ,  $\cos \beta_{12}$  and  $\cos \beta_{23}$ , which are respectively depicted in Fig.9-11. Also it follows from Eq. (39) that the neutrino mass ratios  $|m_1/m_2|$  and  $|m_2/m_3|$  become sensitive functions of  $\xi$ , as are shown in Fig.12 taking typical values of  $r$  and  $\cos \alpha_{12}$ . The common overall factor  $s'/(r'^2\gamma)$  in Eq.(39) is determined to be of order  $10^{-13}$  by the  $\Delta m^2$  data from the solar and atmospheric neutrino oscillation experiments.

Finally we discuss the entries of the CKM quark mixing matrix which are given by Eqs.(16) with (35). In our model, all the elements of the CKM mixing matrix are also

functions of two free parameters  $\cos \alpha_{12}$  and  $\xi$ . The parameters determined so far from the lepton sector do not give rise any inconsistency with the data in quark sector.

## V. SUMMARY

In this paper we have presented and discussed a model of texture four zero quark-lepton mass matrices in the context of  $SO(10)$ . The consistent fitting of the free parameters to the data for neutrino oscillation experiments has forced us to use the charged lepton mass matrix which slightly deviates from purely type I form ( $\xi \sim 0.01$ ). Using this deviated type of mass matrix for the charged leptons and the mass matrices for quarks of type I, we have been able to reproduce four entries in the CKM quark mixing matrix and to predict six entries in the MNS lepton mixing matrix and three Majorana neutrino masses which are consistent with the experimental data. The model has also given the restrictions on the  $CP$  violating phases which came from the neutrino oscillation experiments. Remarkably enough the parameter  $r$  fixed from data fitting is coincident with the value  $r \simeq (2 \sim 3)$  obtained from the renormalization equation [24]. So it is attractive to expect that the above deviation ( $\xi \sim 0.01$ ) from type I form can be obtained by taking the evolution equation of Yukawa coupling fully. (In this paper we have considered the loop correction of gauge boson in the evolution equation.) Though the detail calculations will be developed in the forthcoming paper, we will roughly outline our idea. That is, (charged lepton) mass matrix is exactly of type I at some scale. However, they change their form due to the evolutionary equation of the Yukawa coupling  $Y_a$  until the corresponding Higgs field acquires the vacuum expectation value [28]

$$\frac{dY_a}{dt} = \frac{1}{16\pi^2}(T^f - G^f + H^f) \quad (48)$$

where  $T^f$ ,  $G^f$ ,  $H^f$  are the vertex corrections due to the fermion, the gauge boson and the Higgs boson, respectively. After that, each mass furthermore changes its value according to the mass renormalization equation. The evolution equation of Yukawa coupling is very

sensitive to the Higgs potentials and the initial conditions. One such sensitivity has been found in the behavior of  $\xi$ . The detail will be given in the forthcoming paper.

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# FIGURES

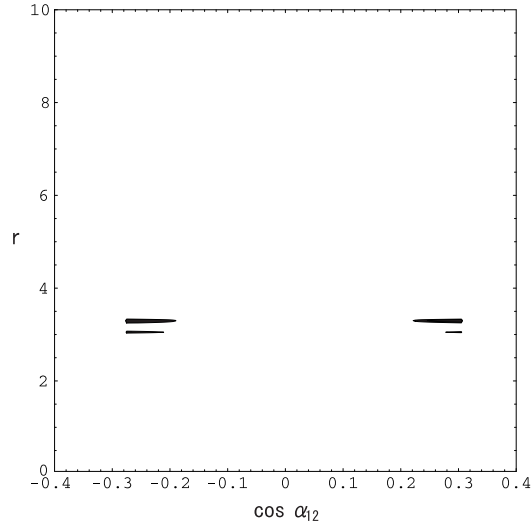


FIG. 1. The case where  $M_u$ ,  $M_d$ , and  $M_e$  are all of purely type I is analyzed. The experimental constraint on  $|U_{23}|$  ( $0.28 \leq |U_{23}|^2 \leq 0.72$ ) gives the allowed region (shaded area) in the  $\cos \alpha_{12}$ - $r$  plane. Here the  $r$  is treated as a free parameter.

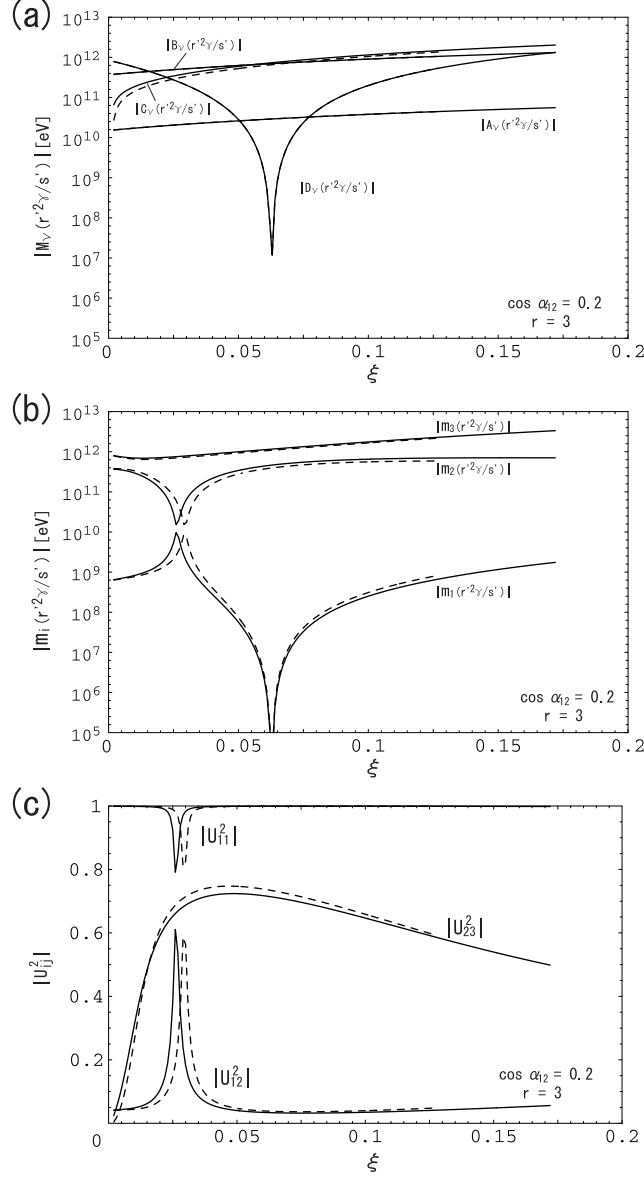


FIG. 2. The slight deviation from type I ( $\xi \neq 0$ ) makes physical parameters change drastically . The dotted lines (solid lines) show the  $\xi$  dependence for  $\cos \alpha_{23} \geq 0$  ( $\cos \alpha_{23} \leq 0$ ) in each diagram. All lines terminate at the points from where  $|\cos \alpha_{23}| \geq 1$  or  $|\cos \beta_{23}| \geq 1$  as will be seen from Fig.9 and Fig.11. (a) The diagram of the elements in the neutrino mass matrix  $M_{\nu}$  versus  $\xi$ . Except for  $|C_{\nu}(r'^2\gamma/s')|$  the dotted lines are overlapped with the corresponding solid lines. (b) The diagram of the neutrino mass eigen values versus  $\xi$ . (c) The MNS mixing matrices versus  $\xi$ .

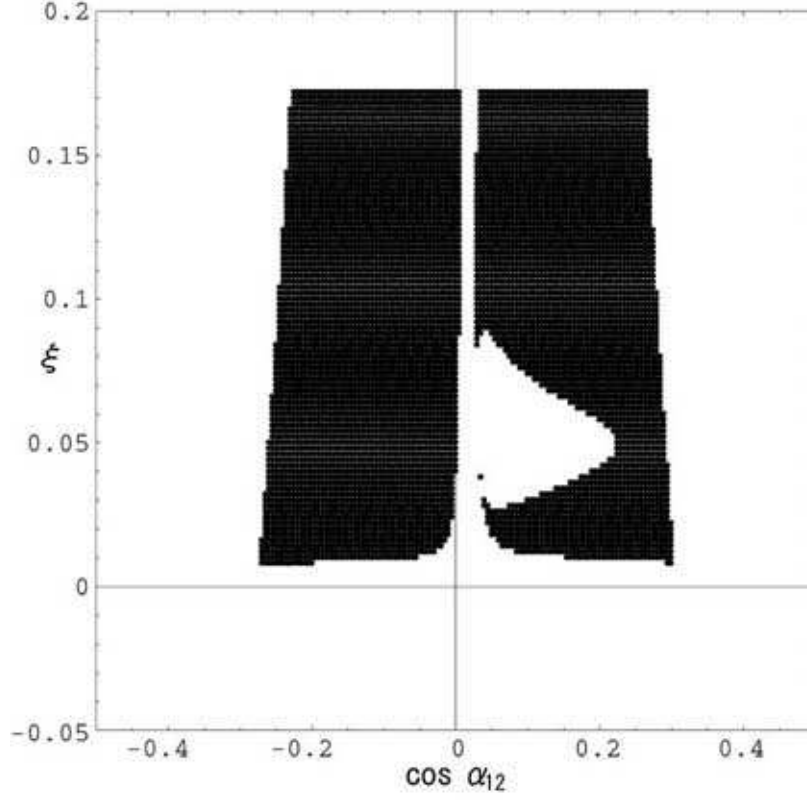


FIG. 3. The experimental constraint on  $|U_{23}|$  ( $0.28 \leq |U_{23}|^2 \leq 0.72$ ) gives the allowed region (dotted area) in the  $\cos \alpha_{12}$ - $\xi$  plane.

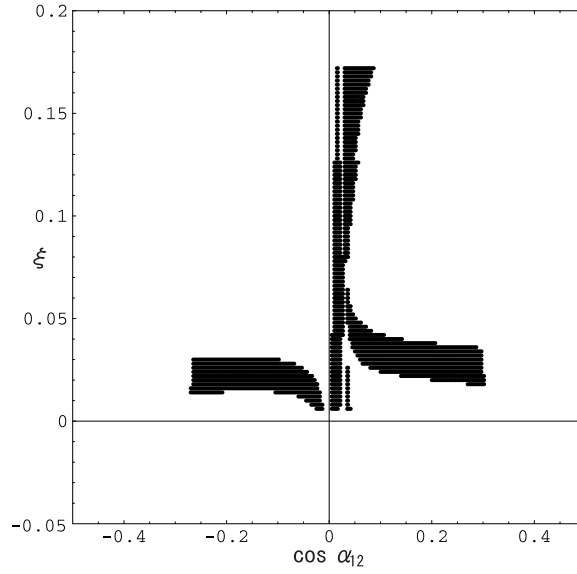


FIG. 4. The allowed region in the  $\cos \alpha_{12}$  -  $\xi$  plane from the experimental constraints  $\Delta m_{12}^2 / \Delta m_{23}^2 \leq (1 \times 10^{-4}) / (3.5 \times 10^{-3}) = 2.9 \times 10^{-2}$ .

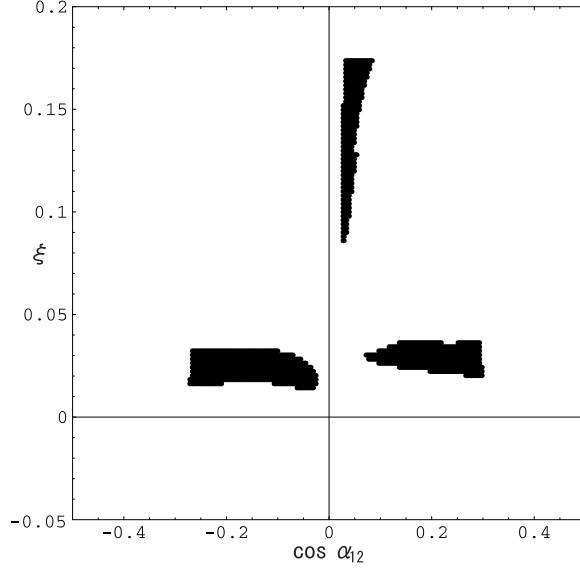


FIG. 5. The allowed region in the  $\cos \alpha_{12}$  -  $\xi$  plane from the experimental constraints  $0.28 \leq |U_{23}|^2 \leq 0.72$  and  $\Delta m_{12}^2 / \Delta m_{23}^2 \leq 2.9 \times 10^{-2}$ .

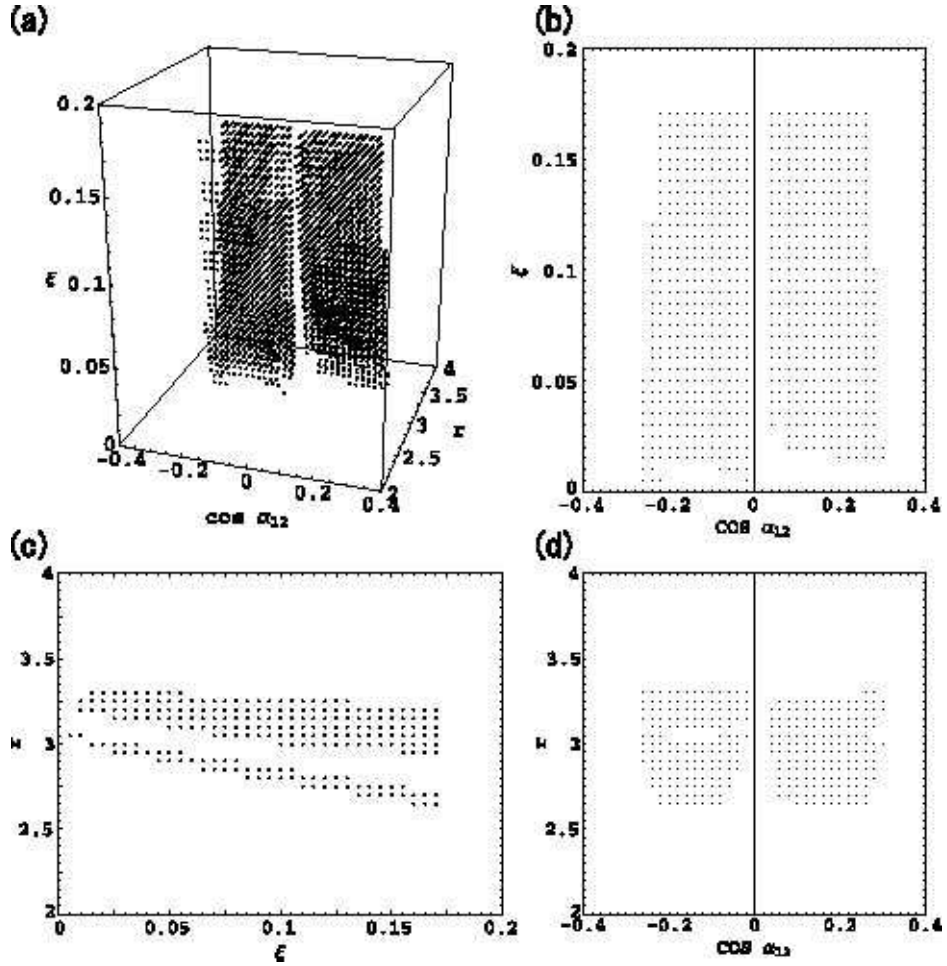


FIG. 6. The  $r$  is treated as a free parameter. (a) shows the allowed region in the  $\cos \alpha_{12} - r$  -  $\xi$  space from the experimental constraints  $0.28 \leq |U_{23}|^2 \leq 0.72$  and  $\Delta m_{12}^2/\Delta m_{23}^2 \leq 2.9 \times 10^{-2}$ . (b), (c) and (d) show the projected allowed regions in the  $\cos \alpha_{12}$ - $\xi$ ,  $\xi$ - $r$ , and  $\cos \alpha_{12}$ - $r$  planes, respectively.

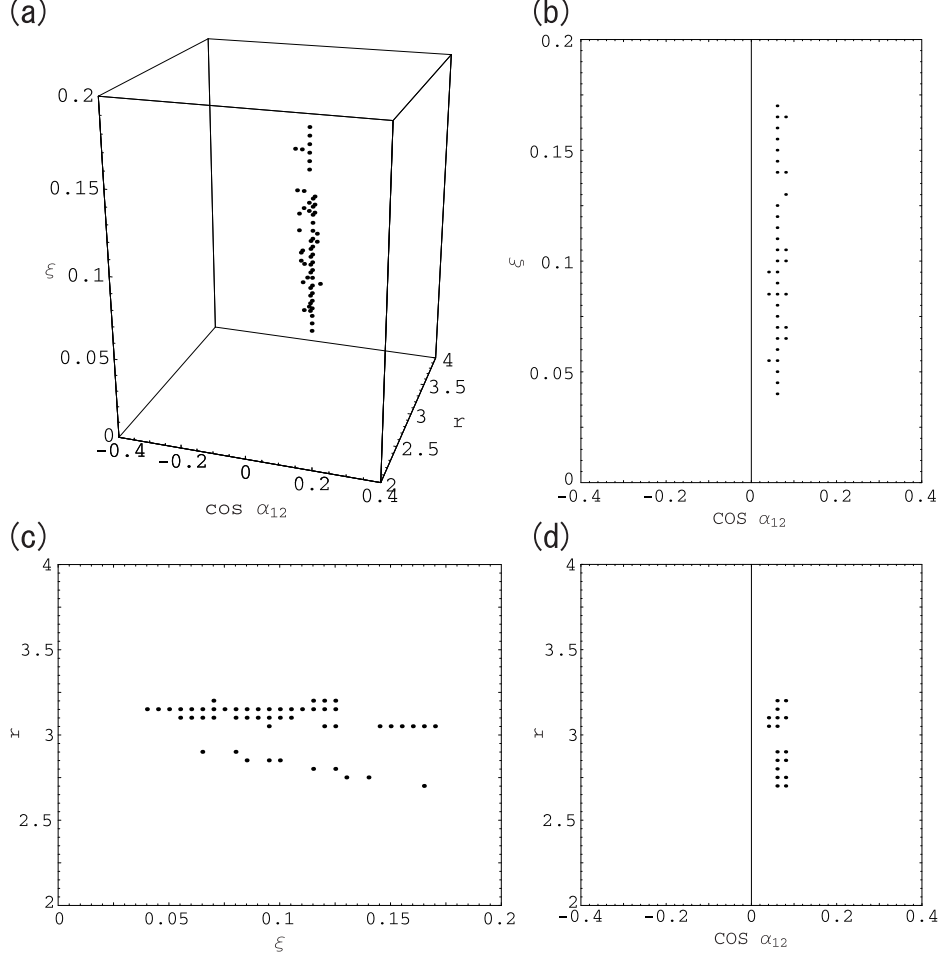


FIG. 7. The allowed region in the  $\cos \alpha_{12}$ - $r$ - $\xi$  space from the experimental constraints  $0.28 \leq |U_{23}|^2 \leq 0.72$ , the small mixing angle solution of the solar neutrino experiments ( $\sin^2 2\theta_{12} = (2 \sim 10) \times 10^{-3}$ ), and the up-to-date value of mass difference  $\Delta m_{12}^2/\Delta m_{23}^2 = ((4 \sim 10) \times 10^{-6})/((1.5 \sim 6) \times 10^{-3}) = (0.67 \sim 6.7) \times 10^{-3}$ .

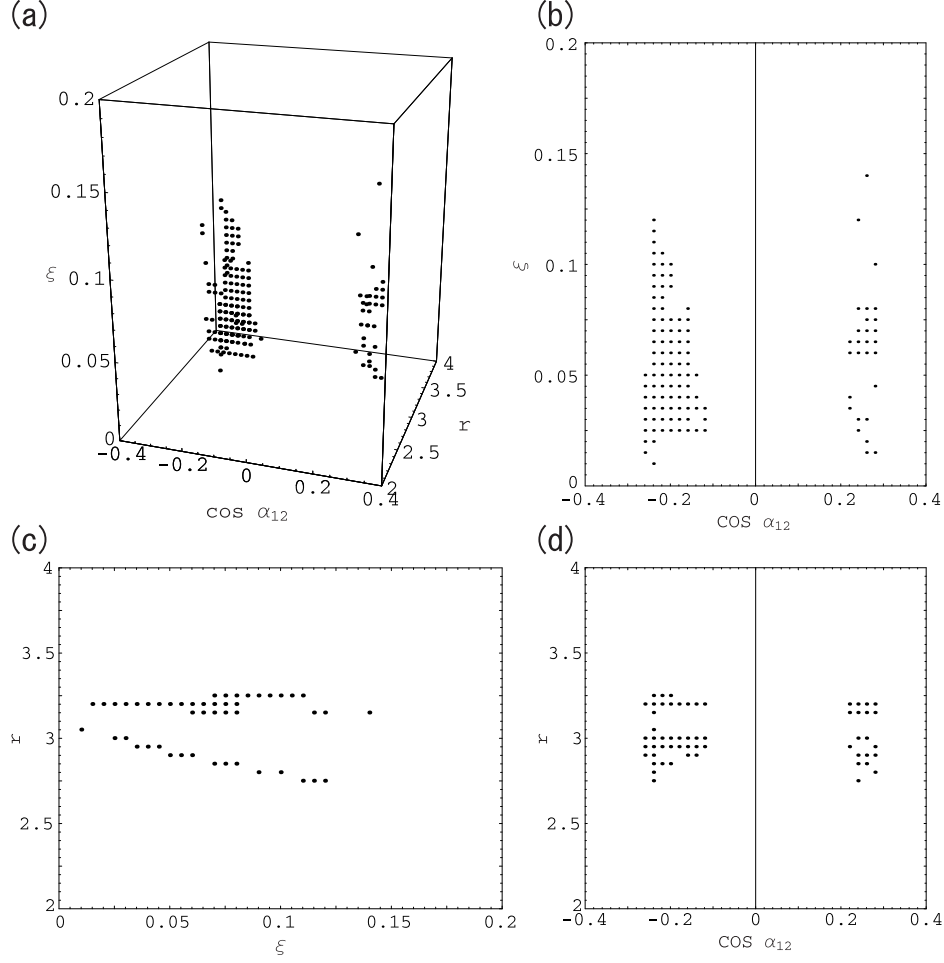


FIG. 8. The allowed region in the  $\cos \alpha_{12}$ - $r$ - $\xi$  space from the experimental constraints  $0.28 \leq |U_{23}|^2 \leq 0.72$ , the large mixing angle solution of the solar neutrino experiments  $\sin^2 2\theta_{12} = (0.5 \sim 1)$ , and the up-to-date value of mass difference  $\Delta m_{12}^2 / \Delta m_{23}^2 = ((8 \sim 30) \times 10^{-6}) / ((1.5 \sim 6) \times 10^{-3}) = (0.13 \sim 2.0) \times 10^{-3}$ .

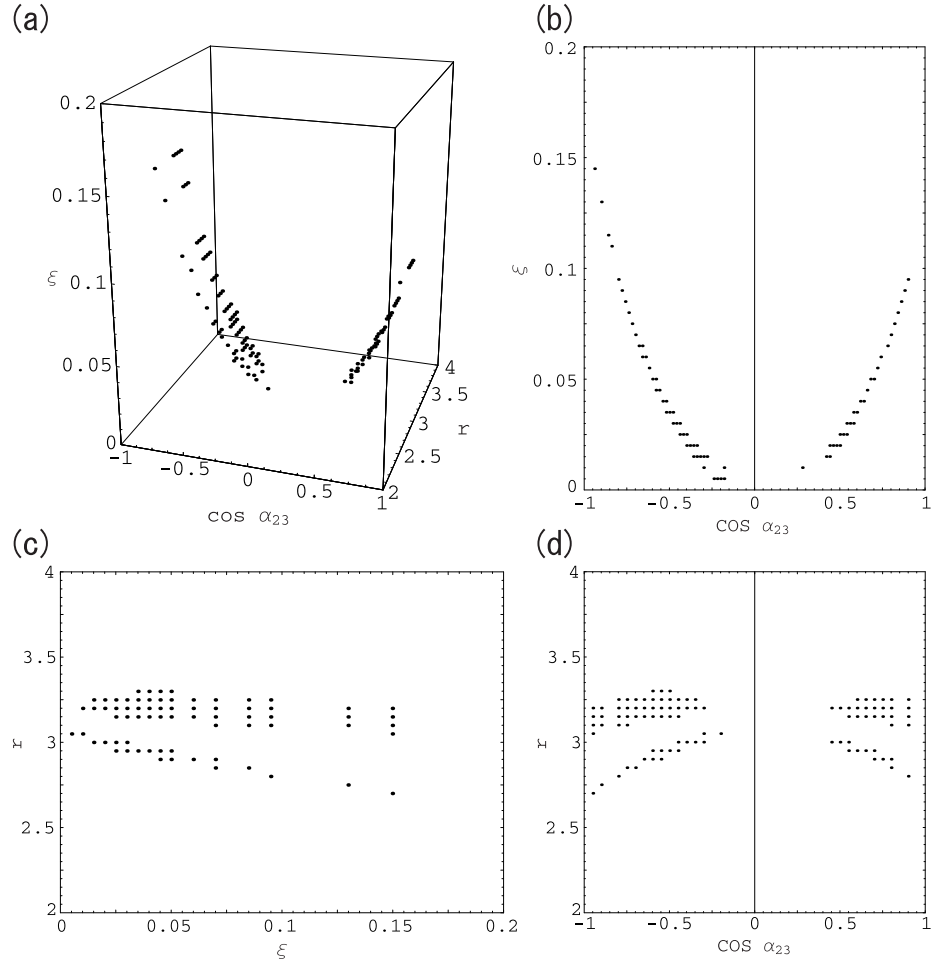


FIG. 9. The allowed region in the  $\cos \alpha_{23}$  -  $r$  -  $\xi$  space from the same constraints as in Fig.6.

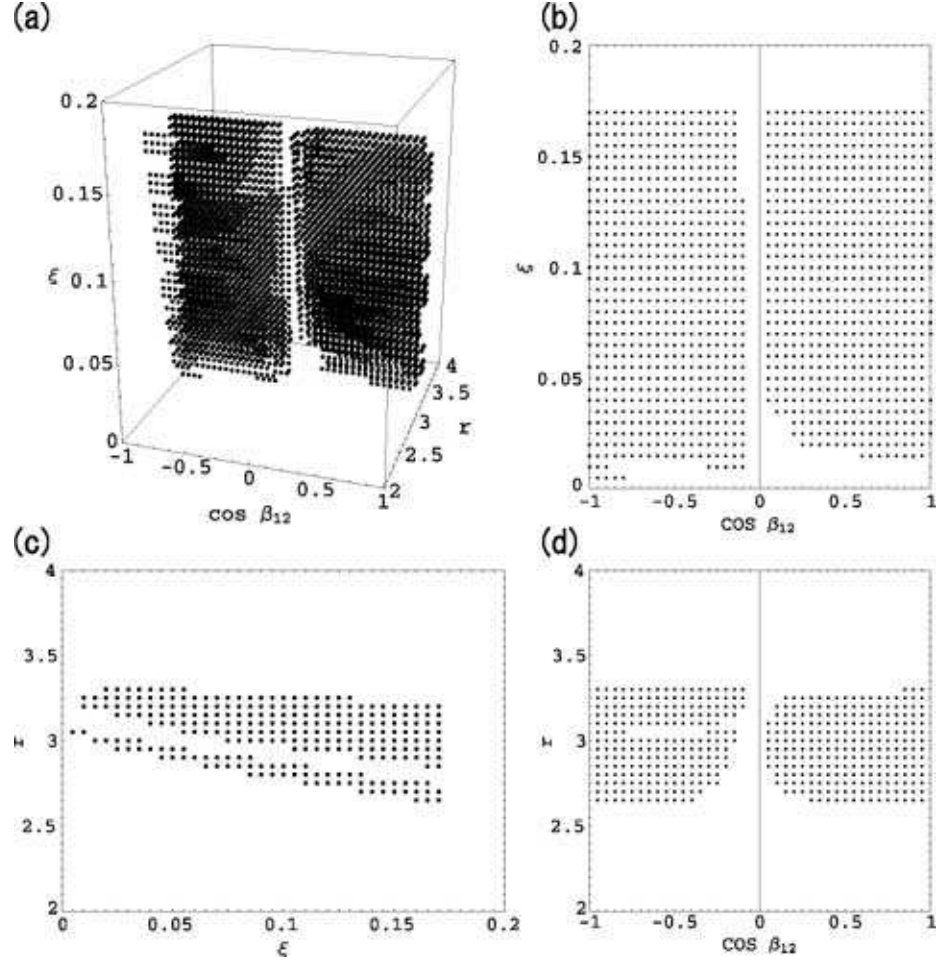


FIG. 10. The allowed region in the  $\cos \beta_{12}$  -  $r$  -  $\xi$  space from the same constraints as in Fig.6.



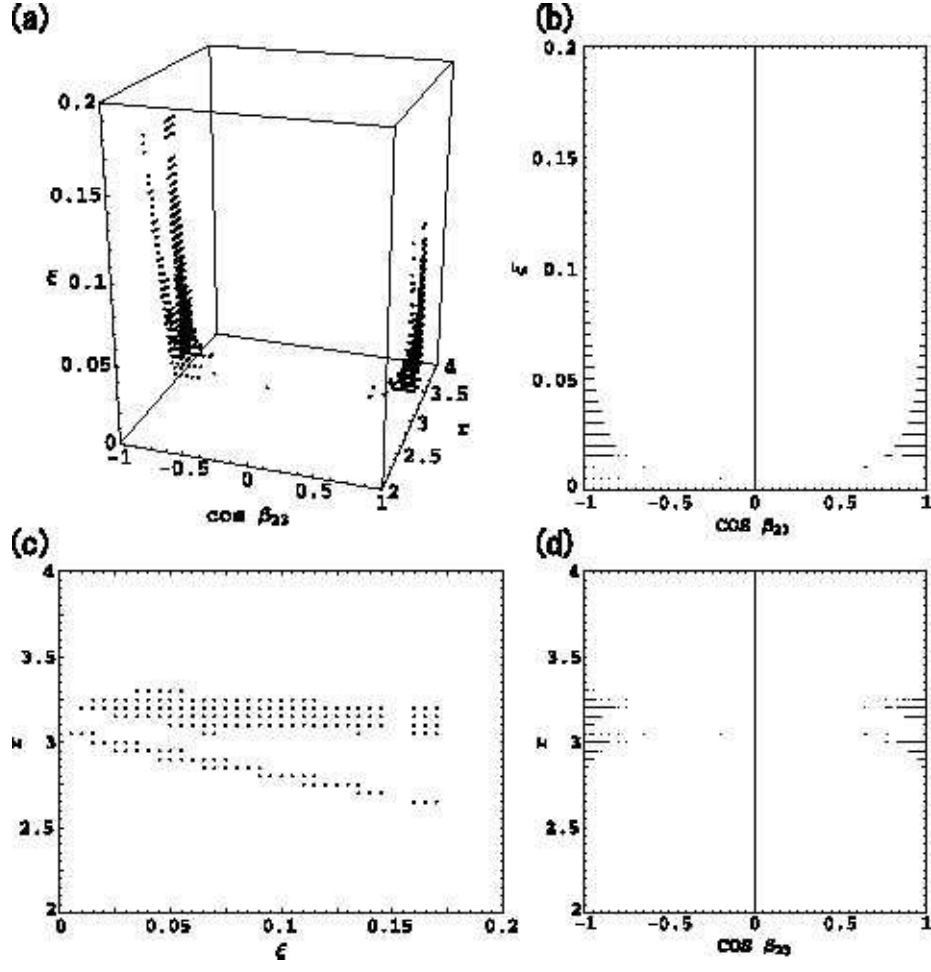


FIG. 11. The allowed region in the  $\cos \beta_{23}$  -  $r$  -  $\xi$  space from the same constraints as in Fig.6.

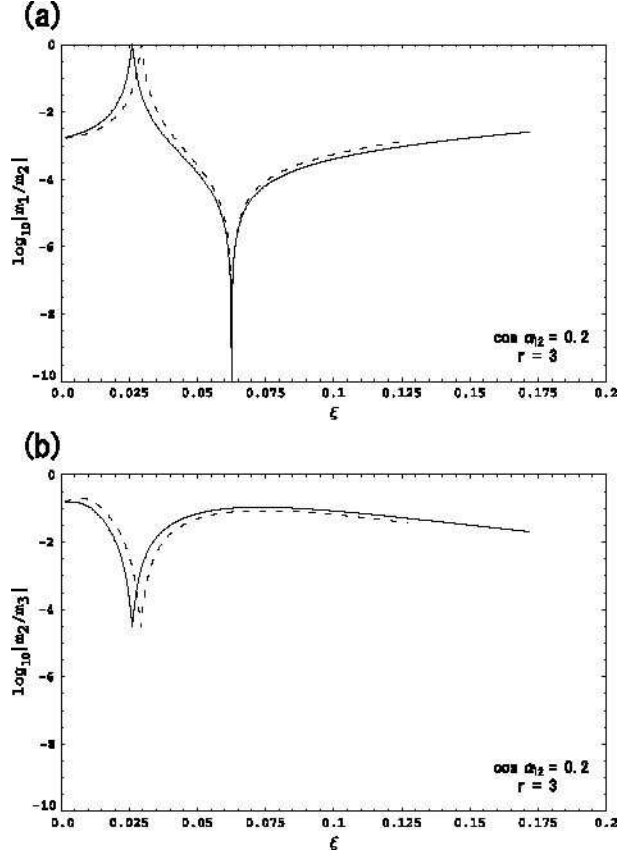


FIG. 12. The dependence of  $\log_{10} |m_1/m_2|$  (a) and  $\log_{10} |m_2/m_3|$  (b) on  $\xi$  for  $\cos \alpha_{12} = 0.2$  and  $r = 3$ . The dotted line (solid line) shows the  $\xi$  dependence for  $\cos \alpha_{23} \geq 0$  ( $\cos \alpha_{23} \leq 0$ ) in each diagram. Both lines terminate at the points from where  $|\cos \alpha_{23}| \geq 1$  or  $|\cos \beta_{23}| \geq 1$  as are seen from Fig.9 and Fig.11. The singular behaviors of (a) and (b) come from those of  $m_1$  and  $m_2$  (see Fig.2).